Determining the Position and Trajectory of Supersonic Projectiles from Acoustic Measurements

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Abstract

Locating the trajectory of an incoming projectile has important military and civilian applications. The origin of the trajectory can be used to locate sniper fire, and the end-point can be used to determine target hit coordinates on appropriately instrumented target ranges used by armed forces as well as civilian agencies (police and sports). Such non-contact determination of hit coordinates can be integrated into a completely computerized instrumentation scoring system.

All supersonic projectiles are accompanied by a bow wave, often referred to as the "Mach Cone" which is virtually attached to the ogive of the projectile and which can be detected acoustically. When a distributed array of microphones detect the arrival of this cone, it is possible to calculate the position and angle of the trajectory from the microphone data.

This paper discusses the analysis and models developed by the author, which enable computation of the trajectory from distributed arrays of acoustic microphones. The mathematical models described here can serve as the basis for an algorithmic procedure, which can be used to obtain the trajectory data in near real time. We also discuss the principal sources of error and the inherent limitations in accuracy of locating trajectories by this method.

Introduction

All supersonic projectiles are accompanied by a bow wave, an approximately hyperbolic shaped shock front, which travels attached to the projectile. The shock wave gradually degrades into a cone shaped acoustic wave-front with a sharp leading edge. This wave is often called the "mach cone" and can be considered as formed by the envelope of the distributed acoustic disturbances from the tip of the moving source (Figure 1). The wave front is well structured and well described as a cone with a half angle, which is only a dependent on the Mach number of the projectile. If the projectile passes near an acoustic

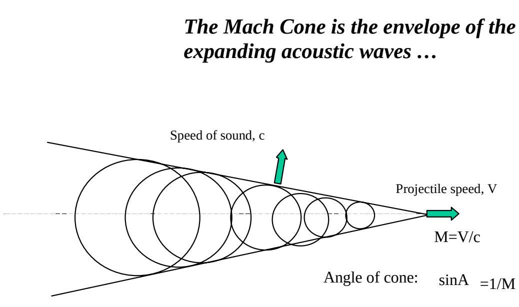

Figure 1

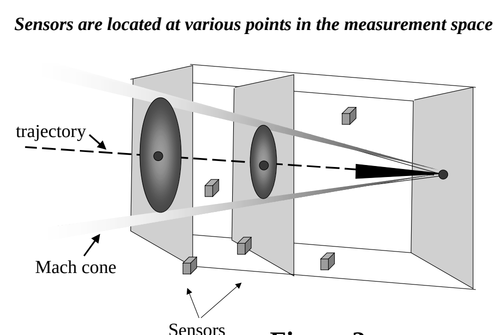

Figure 2

transducer, the pressure jump at the wave front can be easily sensed. This leading edge must be the first acoustic wave to arrive because of the supersonic speed of the projectile. The nearly discontinuous leading edge marks a sharply defined and unique acoustic event. At any point in the wave front, there also exists a well-defined direction

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of travel, which is the propagation vector normal to the wave-front. Two features of this well-defined wave front are therefore measurable: The "time of arrival, TOA", or the "direction of arrival, DOA" of the normal to the front.

As shown in Figure 2, an array of sensors can be distributed in space. The individual sensors nodes can be simple, fast response microphones capable of being triggered by the passing wave, therefore measuring the time (TOA) of arrival at each point. Conversely, each sensor node can be a direction-measuring device, capable of measuring the direction (DOA) of arrival of the incident propagation vector. Methods to measure DOA of an acoustic wave front are discussed later in this paper. If enough sensors are used, each at known locations, it is possible to construct the position and orientation of the cone at any instant from the set of measured TOA's or DOA's. The axis of this cone is coincident with the projectile trajectory and the tip of the cone marks the exact coordinates through which the projectile will pass. It is assumed that the sensors are distributed in a sufficiently small spatial volume, such that the trajectory is this vicinity is well approximated as a straight line. The projectile is assumed traveling at constant speed with zero angle of attack so that the axis of the trajectory and the axis of the cone are coincident.

Analytical Approach

In general, the sensors can be distributed in arbitrary locations in the measurement space, providing that the position coordinates of each sensor node are known. Although the analysis conducted here is carried out for the general case, it is perhaps easier to envision the solution if we assume that the sensors are placed in regular positions in the coordinate system, for example, constrained to planes normal to the trajectory. Although such placements of sensors are easier to visualize in solving the problem, we do not know in advance the precise arrival direction, nor is it always easy to place the sensors in pre-fixed positions. For these reasons it is important that we are able to solve the problem for the general, rather than for the specific case. The general analytical method is described here, but for ease of presentation, sample calculations are shown for simple cases.

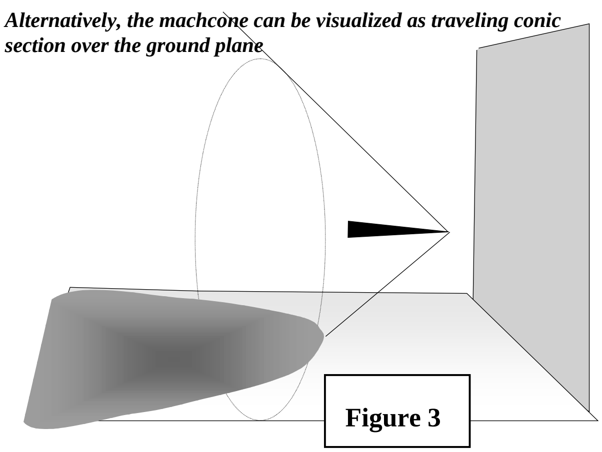


Figure 3

Some simple case visualization's can lead to certain simpler, but less general, analyses. For example, the traveling mach cone can be envisioned either by the intersection of the cone with vertical planes, perpendicular to a horizontal trajectory, or by the intersection of the cone with a horizontal, ground plane. Since the intersection of a cone and a plane is always a conic section, the locus of points is either that of an expanding ellipse/circle in the vertical plane or that of a traveling hyperbola on the horizontal plane. (Figures 2,3) Either geometry can lead to special case solutions, but for purposes of this paper, simplified models will be developed based on the expanding ellipse/circle model. These special case configurations give a great deal of insight to understanding the analysis and to estimating principle sources of the error budget, but they lack generality. The generalized solution methodology developed here leads to more powerful, but less easily visualized, algorithmic methods suitable for implementation in the 3D case.

Time Difference of Arrival Solutions (TDOA)

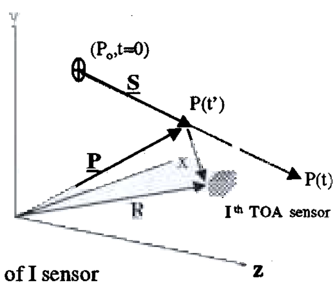
In order to measure TOA of the incident wave at each sensor in absolute terms, it would be necessary to have the time synchronized with the time of arrival of the projectile itself. If it were possible to set time zero at the instant the projectile crossed into the measurement space, then knowing absolute time of arrival at each sensor would greatly simplify the problem. This is not possible. Accordingly, the best we have to work with are the differences in TOA at each sensor. Only when the first sensor is triggered can we know the beginning of the events. By comparing the trigger time at all other sensors with one particular sensor, we obtain a set of measurement data, which are really time differences of arrival, (TDOA)'s. The problem must be solved in terms of TDOA data, not TOA data.

Generalized Time Difference of Arrival Solution:

With reference to Figure 4, the projectile is assumed to enter the measurement space at unknown time, t=0. The general 3D solution below is developed by vector-matrix methods by which multiple scalar equations can be reduced to only two compactly stated vector equations. The trajectory vector is defined by the speed and heading of the projectile constituting the velocity vector $\vec{V}$. The unit vector of the trajectory, $\hat{\mathbf{s}}$, can be given either by three direction cosines or by azimuth and elevation of the trajectory vector. The position of the projectile, $\vec{P}(x, y, z)$, is defined by the position coordinates of any particular point on the trajectory, say the initial entry coordinates at time zero. The trajectory to be determined is therefore the 6-vector: $(x_o, y_o, z_o, V, s_x, s_y)$ or more compactly, $(\vec{P}_o, \vec{V})$. Once this vector is determined, the position of the projectile at any time is known since the trajectory is assumed a straight line in this portion of the measurement space.

The mach cone arrives by the ***shortest*** *acoustic time from P(t') to the sensor*

Figure 4



R$_i$: Position vector of I sensor
P: Position vector of projectile
S: Trajectory Vector

Because the projectile is supersonic, the acoustic wave front is conical in shape and propagates with the speed of sound, c, normal to its surface. In accordance with that concept, the time of arrival of a sound wave at any sensor, $\vec{R}_i$ which originated at the point, s, will be given by the vector equation

$$t_i = \frac{s}{V} + \frac{1}{c}\left|\vec{P}(s) - \vec{R}_i\right| \qquad (1.1)$$

Where t_i is the time of arrival at the ith sensor, V is speed of projectile, c is speed of sound, and $\left|\vec{P}(s) - \vec{R}_i\right|$ is the distance from the emanating point, s, to the ith sensor at position $R_i(x, y, z)$

The time of arrival is a function of s along the trajectory. We recognize that we are looking for the first arrival of the acoustic wave front. The value of s that minimizes the time of arrival function will be a unique point on the trajectory from which the first wave front arrives at the sensor. All other points will generate a signal which takes longer to arrive and will not constitute the leading edge of the front. The value of s that minimizes t_i can be found by differentiating eqtn 1.1, setting it to zero, and solving for the value of s_o. Carrying out this process results in:

$$\frac{1}{V} + \frac{1}{c}\left[\frac{\left[\hat{s}\cdot\left(\vec{P}(s) - \vec{R}_i\right)\right]}{\left|\vec{P}(s) - \vec{R}_i\right|}\right] = 0 \qquad (1.2)$$

Where $\hat{s}$ is the unit trajectory vector. The value of s that satisfies the above equation is unique for each sensor. V/c is the Mach number, M, of the projectile. The quantity in parenthesis is simply the dot product of the trajectory unit vector, $\hat{s}$, and the unit vector pointing from the projectile at s to any sensor. It is obvious that, for any sensor, the cosine of the angle between the trajectory vector and the line of sight to any sensor must always be equal to 1/M when the wave arrives. Since the sine of the half angle of a mach cone is given by 1/M, the angle between the projectile axis and line to any sensor must be the complementary angle of the mach half angle. This would be expected by examination of the geometry. The conditions set by equations 1.1 and 1.2 above are sufficient to be able to compute the unknown trajectory vector from the times of arrivals of the first wave at each sensor, if enough sensors are used.

As stated above, the actual time the projectile enters the space, time zero, is not known. We can eliminate t_0 from the equations above by referencing all times to one sensor, say the first to be triggered. The measured times of arrival are therefore converted to time differences of arrival, relative to any one of the sensors. Accordingly we make the substitution: $\tau_{i,j} = t_i - t_j$, where the τ 's are the TDOA's, the actually measured data.

Expanding the above expressions provides a sufficient number of independent equations if enough sensors are used to compute all of the unknowns. Definition of the trajectory requires solving for 6 unknowns. If the speed of sound is not known by separate measurement, this also becomes a seventh unknown. Accordingly if a minimum of 7 sensors is used, it is possible to solve them simultaneously to find the 6 unknowns of the trajectory (including the speed of sound, c) in terms of the known positions of all the sensors ($\vec{R}_i$) and the measured differences in times of arrival ($\tau_{i,j}$). Several well-known methods are available to solve systems of non-linear, vector equations, such as Newton's method, Conjugate Gradient method, Levenberg-Marquardt algorithm, etc.

Simplified Illustration of TDOA Solution

By restricting the generalized 3D-vector analysis above to a single 2D plane, a simplified set of calculations can be obtained, at the expense of generality, by making simplifying approximations.

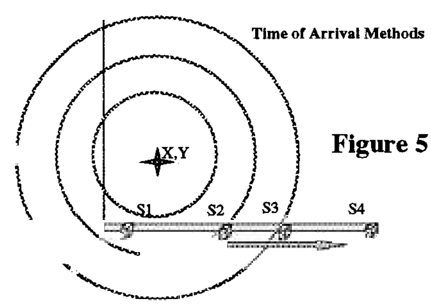


Figure 5

The expanding circle, eventually crosses the sensors at time, t
The center, x,y can be computed from the measured arrival times

Assume that the projectile is normally incident on an unknown location on a vertical plane. Assume that the TDOA sensors are arranged at various points throughout the plane. We wish to find the location coordinates (x_o, y_o) of the projectile penetration from TDOA measurements at the sensors. This will not provide the direction of the trajectory, but if the process were repeated at a second plane a known distance behind the first plane, a second set of coordinates would be obtained by the same procedure. Two sets of 2D coordinates at known spacing are sufficient to fully compute the direction of the trajectory, which connects them.

In this case, the locus of points representing the intersection of the mach cone with the plane is that of an expanding circle centered about the penetration point (Figure 5). The expansion rate of the circle is given by

$$cw = \frac{c}{\sqrt{1 - 1/M^2}}$$

Where cw is the speed of expansion of the pseudowave in the plane, c is speed of sound in air; M is the projectile mach number. Note that the pseudo-wave is attached to the physical wave, but does not travel at the same speed.

The time of arrival of the circle at any point, x, on the axis is given by:

$$t(x, y, cw) = \frac{\sqrt{(x - x_o)^2 + (y - y_o)^2}}{cw}$$

This enables us to compute the times of arrival at each of several sensors and known x locations on the axis. The TDOA's are obtained by subtracting any two TOA's from any pair of sensors. This equation has 3 unknowns since we do not know the M of the projectile, therefore the cw. Use of four sensors generates a sufficient number of non-linear TDOA equations to simultaneously solve for the unknowns

Clearly this is an oversimplification. In general, the projectile will not be normally incident on the planes. The pseudo-wave then converts to an ellipse, which is concurrently expanding about a walking center. Further, the speed of expansion will become a function of position on the ellipse. Although these equations can also be developed and solved the process becomes burdensome, and the power of the general method described earlier becomes appreciated.

Direction of Arrival Solutions (DOA)

With reference to Figure 6, the projectile enters the measurement space at an unknown location and with an unknown velocity vector (speed and direction). Because the speed of the projectile and/or the speed of sound is unknown, the Mach number, M, of the projectile is initially unknown. The situation is exactly the same as described in the TOA section of this paper, but this time an array of DOA sensors is used instead of an array of TOA sensors. A DOA sensor is capable of measuring the direction, not the time, of the incident wave front. The direction of the wave front is given by the wave propagation unit vector, $\hat{\mathbf{k}}$, consisting of three direction cosines of which two are independent. Direction of a vector in 3D space is fully specified with two numbers (or 2 angles)

DOA Sensors

A single, "point" transducer is generally not capable of measuring direction. For reasons inherent in the physics, the measurement of "direction" requires "spatial aperture". Many types of DOA sensors have been devised but they all require some physical aperture either directly or indirectly. A common configuration for a 3D DOA sensor is to arrange at least 3 microphones in a plane, with a small physical spacing between them. The spacing constitutes the spatial aperture, and direction is measured by comparing the differential arrival times between the three closely spaced sensors. Implicit in this concept is that the direction is the same at all three sensors, thus the incident wave must be planar or sufficiently far from the origin of the source to meet the planar approximation. Since "direction" is actually obtained from "time' measurements, the precision of direction will also depend on the precision with which time can be measured, hence bandwidth (temporal aperture) is also important. It can be shown that the larger the physical aperture and the larger the bandwidth, the better will be the angular precision with which the direction can be measured. Regardless of the design details, the device generates two numbers, which are a measure of the two direction cosines, or alternatively, two angles (azimuth and elevation) of the incident ray vector.

2D DOA sensors are designed to measure only one component of the incident direction, which lies in a particular plane. Ideally, the non-coplanar component of the incident propagation vector should not interfere with the desired in-plane component, otherwise cross-talk can limit the precision of measurement of the desired in-plane component.

This brief description of DOA sensors will suffice, as it is not the purpose of this paper to provide a detailed discussion of the analysis of DOA design.

Two DOA Sensors and the Trajectory form a 3D tetrahedron, which can be Solved if we measure k_1 and k_2 and the Mach Angle, μ …

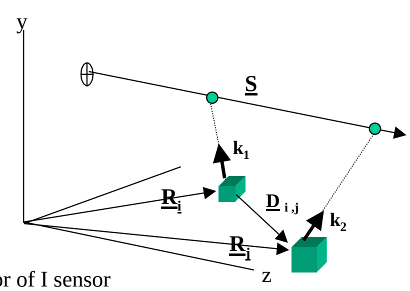


Figure 6

$\underline{\mathbf{R}}_i$ Position vector of I sensor
$\underline{\mathbf{R}}_j$ Position vector of j sensor
$\underline{\mathbf{S}}$: Trajectory Vector

Generalized Direction of Arrival (DOA) Solution

As discussed previously, the arrival of the first acoustic wave is also the arrival of the leading edge of the "mach cone" wave front. Because the mach cone has a well-defined angle, and since the propagation vector points normal to the wave front (in still air), all the shock waves leaving the projectile have a propagation vector that always leaves the projectile axis at an angle of α, which must be the complementary angle of the half-cone, μ. This condition can be expressed by the following equation, which must be true for each DOA sensor:

$$\hat{k}_i \cdot \hat{s} = 1/M$$

Where, $\hat{\mathbf{k}}_i$, is the unit propagation vector at the i[th] sensor, and $\hat{\mathbf{s}}$ is the unit trajectory vector. This defines one scalar equation for each DOA sensor used.

The second observation is that, for any two DOA sensors and for any two points on the trajectory, we have 4 non-coplanar points (generally) in a 3D space. These 4 points form a solid tetrahedron. The exception to this case will occur only if the trajectory lies in the same plane as the line joining the two sensors. Analogous to the 2D process of "triangulation" (in which the total triangle is solved from knowledge of its parts), we define a process of "tetrahedralization". In this process, the entire tetrahedron is solved from knowledge of some of its sides and angles. Once solved, all of the sides and angles are known, thus defining the trajectory. To solve the tetrahedron we simply use the fact that the vector sum of the sides of any closed figure must always equal zero. For our case, this equation is written as:

$$\vec{L}_i + \vec{S} = \vec{D}_{i,j} + \vec{L}_j$$

$$\vec{D}_{i,j} = \vec{R}_j - \vec{R}_i$$

Where $\vec{L}_i$ is the vector side from sensor i to a point on trajectory, $\vec{S}$ is the vector distance between the two points on the trajectory, $\vec{D}_{i,j}$ is the vector connecting a pair of DOA sensors, and $\vec{L}_j$ is the vector side from sensor j to the second point on the trajectory.

From 2.1 above it is evident that the vectors, L, connecting the sensors to the trajectory are actually rays of the mach cone wave front and they must all form the same angle with the trajectory axis. The cosine of this angle, α, must be given by 1/M. Vector equations in 2.2 will form 3 scalar equations for each possible pair of DOA sensors. Expanding the above two equations results in a sufficient number of independent scalar equations to solve for the unknowns, which collectively define the Trajectory State Vector: $\vec{\mathbf{T}}(x, y, z, \theta, \phi, M)$. Note that in the DOA case, the projectile state vector is written in terms of the mach number, M, rather than the speed, V. The DOA solution does depend directly on knowing the speed of sound in air. It is only dependent on the geometry of the incident propagation vector, which depends only on M.

Manipulation of the above fundamental equations result in developing a set of simultaneous vector equations containing the unknown trajectory vector as an intrinsic variable along with the known sensor position matrices and measured DOA matrices. The equations can be solved to extract the unknown vector by methods, which are straightforward, but the detail will not be repeated here for brevity.

In abbreviated vector notation, the overall algorithm can be expressed as a vector function of matrix arguments:

$$\vec{\mathbf{T}} = \mathbf{f}(\overline{\overline{\mathbf{D}}}, \overline{\overline{\mathbf{k}}}) \qquad \text{where,}$$

$$\overline{\overline{\mathbf{D}}} = \left[\vec{\mathbf{D}}_1, \vec{\mathbf{D}}_2, \vec{\mathbf{D}}_3\right] \quad \text{the position matrix of sensors}$$

$$\overline{\overline{\mathbf{k}}} = \left[\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2, \hat{\mathbf{k}}_3\right] \quad \text{the DOA matrix at all sensors}$$

If the mach number of the projectile were known it would be possible to determine the trajectory with only 2 DOA sensors. Because it is not known in advance, a third DOA sensor will result in an over-determined system of equations. Keeping in mind that each 3D DOA sensor contains 3 individual acoustic transducers, hence the system is actually using a total of 9 transducers.

Simplified Illustration of DOA

As discussed for the TOA case, By restricting the generalized 3D vector analysis above to a single 2D plane, a simplified set of calculations can be obtained, at the expense of generality, by making simplifying approximations. Again, we assume the projectile is normally incident at an unknown location on a vertical plane. At the base of the

plane, we place two or more DOA sensors. The locus of the mach cone will again be an expanding circle, but this time we focus on the direction of travel of the wave front, which in general will not lie in this plane except for extremely high mach numbers. Nevertheless, by neglecting the out of plane component, the in-plane propagation vectors point to the origin of the circle term. If two angles to a source are known, the source position can be computed by simple triangulation (Figure 7). If the process is repeated at a second plane, then the direction of the trajectory is computed from the two sets of positions.

If two DOA angles are known in a plane
The center can be found by triangulation…

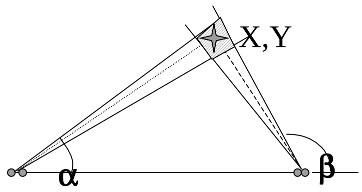


Figure 7

In this case the source coordinates are given by

$$x(\alpha,\beta) = \frac{X_A \tan(\alpha) - X_B \tan(\beta)}{\tan(\alpha) - \tan(\beta)}$$

$$x(\alpha,\beta) = \frac{\tan(\alpha)\tan(\beta)\left[X_A - X_B\right]}{\tan(\alpha) - \tan(\beta)}$$

Where the two DOA sensors are located at X_A and X_B and the two measured direction angles are α and β.

For the this case, the coordinates of the projectile are expressed in terms of the measured angles and it is not necessary to solve simultaneous vector equations.

Error Sensitivity Analysis

The accuracy with which projectile location and direction can be measured by the above techniques is limited by a wide variety of parameters in the overall error budget. The major sources of error can be classified as due to (a) instrumentation error, (b) obliquity and geometry error, (c) atmospheric propagation error, and (d) clutter error.

Accuracy consists of bias error together with precision error. Constant bias errors can be calibrated out and are not true errors. Random bias errors and precision errors are stochastic processes, which cannot be fully corrected for because they are unpredictable. Although accuracy, precision, and resolution are not the same things, as a rule of thumb with well-calibrated instrumentation, accuracy is limited by the same random variables that limit precision and resolution. For purposes of this paper, we consider that the accuracy will be limited by, and of the same level of magnitude, as the precision and resolution of the measured observable. With due respect to the differences, these terms will be used interchangeably.

Instrumentation Error: TDOA systems will be limited by the precision/ resolution with which the time of arrival of a signal can be measured. Ultimately, in terms of the basic physics, the TDOA capability will be directly related to the "temporal bandwidth" of the microphone-electronic subsystem relative to that of the measured signal. Along with bandwidth the available signal to noise ratio will be the ultimate limitation in ability to measure difference in TOA of two closely spaced waveforms. For DOA systems, we are similarly concerned with the precision/resolution with which the direction of an incident vector can be measured. For DOA concepts, similar physics considerations lead us to be limited by the "spatial bandwidth" (physical acceptance aperture) relative to the incident spatial frequency content of the wave front shape. Since DOA is generally measured from time differentials, bandwidth is still an important factor. Similarly, the available signal to noise ratio will ultimately determine the ability to resolve two closely spaced direction vectors. The most important metrics for the sensor nodes are their impulse response (convertible to frequency response) and the aperture size over which the direction sensing aperture is distributed. Internal instrumentation noise will serve as an eventual limiting factor for all such systems. Given these specifications, the ultimate precision and resolution with which we can measure time and direction can be estimated.

Obliquity and Geometry Error: Certain approximations can be made to simplify the computations necessary for a full and proper solution. In simplified cases, the sensors are arranged in regular planes facing the anticipated angle of incidence of the projectile. For these cases, the computational solutions are easier and quicker, but when actual projectiles deviate from the assumed idealizations, errors are introduced. For example, an obliquely incident projectile does not cross the instrumented planes in the same way, idealized circular cross sections, become elliptical, etc. Another source of error is because the bow wave only approximates the shock cone at some distance from the projectile. For small caliber rounds, this is a good model, but for large, especially blunt, caliber rounds, the

leading edge of the bow wave is neither conical nor attached to the projectile ogive. When such a projectile passes too near any sensor, this departure of the shape of the wave from the idealized mach cone can cause a geometrical error, which we refer to as the “bow wave effect”. Unless properly accounted for, such geometric variations from the idealized design model will result in errors.

Propagation Error: Calculation of the mach cone center is dependent on the assumption that the mach cone is well defined and does not deviate in shape over time. The shape and angle of the mach cone is, in turn, dictated primarily by the speed of sound in air and the speed of the projectile. For stationary, homogeneous air masses, this is a good idealization since the speed of sound is the same throughout the medium. If the acoustic wave front becomes distorted in transit to the sensors, the original assumptions will be invalidated and errors will occur in the calculated position of the mach cone axis. The speed of sound will be dictated by two primary factors: the temperature of the air and the velocity of the windstream when the air is in motion. Air composition, i.e. humidity, is also a factor but these effects are neglected. The major drivers for this error will be variations in air temperature and in wind conditions. This will be especially true when the atmospheric winds are inhomogeneous and turbulent, containing internal gradient distributions of temperature and/or wind. Gradients will become especially pronounced near the surface of the earth in high wind and/or high temperature as discussed below.

Clutter Error: This constitutes all sources of error due to the arrival of externally originating, unwanted, acoustic interference of any type. This includes wind induced microphonic noise, indirect acoustic propagation paths, general intense ambient noise, battlefield sounds due to multiple round firings, and even negative clutter i.e. acoustic shadow zones formed by nearby objects. In general, most of this noise can be reduced or eliminated by employing temporal frequency responses designed to eliminate most noise sources while passing the essential high frequency portions of the shock wave front. Since we are dealing with supersonic projectiles, the mach cone will always be the “first” wave to arrive (via an open air path) at any sensor, thus eliminating most air-based echo’s as secondary arrival signals. A note of caution is in order here. Since in many cases the projectile may strike a solid object, which in turn is acoustically coupled to a sensing microphone, the propagation speed through the solid coupling will usually exceed the speed of sound in air, thus the acoustic signal from such paths can arrive at the sensor in advance of the true bow wave. Special care must be taken to assure that the sensor sees only the open air paths from the projectile, rather than such secondary paths.

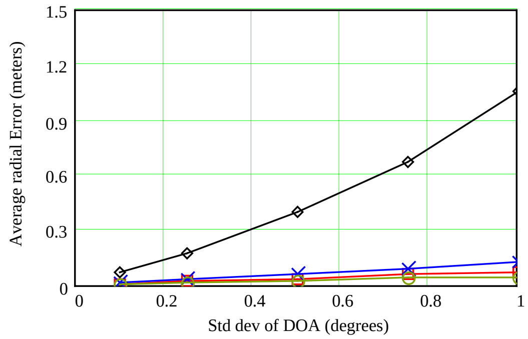
Sensitivity Analysis

Assuming that the clutter interference is controlled and/or properly filtered, and assuming that the geometry of the mach cones do not deviate markedly from their idealized forms, we determine the major limitations to accuracy of projectile location are those of instrumentation error and propagation error. Of these, the ultimate limitation will be that of propagation error through real world atmospheres. This occurs because the acoustic wave front becomes increasingly distorted as the path length increases due to atmospheric effects of wind, turbulence and temperature. The stochastic effects of gradients and movements of the air are not controllable or easily correctable.

Sensitivity to Instrumentation Error

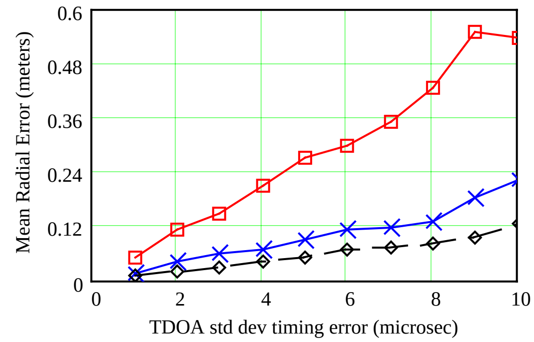
For TDOA systems, the location accuracy will be a direct function of the timing resolution with which it is possible to separate arriving waveforms. For DOA systems, the location accuracy is similarly a function of the precision with which it is possible to measure incident direction angles. To determine sensitivity of either of these systems, to timing error or to angular error as the case may be, the models described above were used to simulate errors in position from errors in time or direction, by Monte Carlo methods.

In the case studies here, we assume that we are attempting to locate the position of an incoming projectile over an approximately 5 X 5-meter plane with either TOA or DOA sensors. The measurement errors are normally distributed about their true values with various standard deviations. For TDOA systems, the standard deviation of error is in microseconds. For DOA systems, the standard deviation of error is in degrees. The errors in measured positions were computed in terms of their means and standard deviations in both x, y, and radial coordinates. Figures 8 and 9 are indicative of typical spreads in radial position error as a function of measured time or direction errors respectively. The errors will vary with location and distance of the projectile from the assumed sensor positions. Each trace represents a zone of impact in the plane, such as center, lower left, top center, etc. In figure 8, the largest errors occur when the projectile passes too near the sensor baseline. In figure 9, the largest errors occur



Radial Position Error vs DOA angle error

Figure 8



Mean Position Precision vs TDOA error

Figure 9

at points near the center of the plane where the TDOA was nearly zero at all sensors. Examination of the scatter diagrams (not shown here) also shows that the error distribution is by no means symmetrical in x and y. For example, the errors in the TDOA system of figure 9 were strongly dominated by large y error and small x error. The results shown here are only intended to be typical for certain arrangements of the sensors, but the results will vary greatly for different geometrical distributions of the sensors relative to the measurement space and its relationship to the incoming round geometry.

Sensitivity to Propagation Error

Errors due to atmospheric effects will be only briefly discussed here in qualitative terms due to the complexity of the phenomena.

The models and analyses above assumed that the projectile was locatable because of the assumed integrity of a well-behaved mach cone shaped bow wave. For quiet and homogeneous air, this is quite true and the projectile can be located accurately for well-designed, instrumentation-limited systems.

For uniform crosswinds, the mach cone will tilt relative to the projectile axis, but will not change shape. Because of the high speed of the projectile, relative to the wind, this effect is negligible compared to instrumentation error and can be corrected for if necessary with associated anemometer readings. Wind and temperature gradients, on the other hand, result in distortion of the shape of the wave fronts. (Fig. 10).

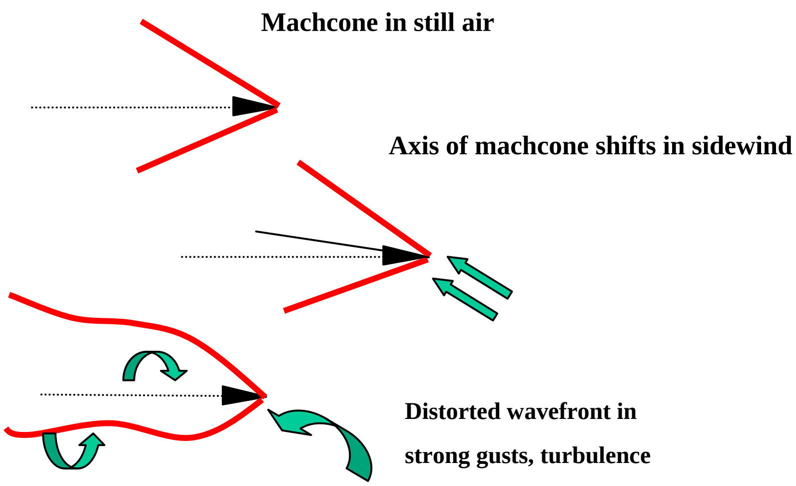


Figure 10

Unfortunately, projectile measuring instrumentation of this type will usually by mounted at or near the surface of the earth for practical reasons. Wind and temperature distributions can follow many models but it is precisely in the surface boundary layer, within the last few meters, that the air sustains extremely wide variations in turbulence, temperature, and wind extremes. A review of the literature dealing with microclimatology yields accounts of measurements of temperature and winds near the earth surface. Temperatures can fluctuate 5 to 10 degrees centigrade in a matter of seconds, and winds can fluctuate on the order of 30% of mean value in turbulent gusts and eddies. The fluctuation levels will depend on season, time of day, geographic location, solar loading severity, upper air wind severity, etc. These transient gradients, most severe near the ground surface, can result in substantial refraction of

acoustic waves. Stochastic and dynamic refractions cause the rays to change direction and the conical wave shape to change form as they approach the surface of the earth. DOA systems can only determine the actual direction of the incident wave, but when refracted, these rays point to erroneous virtual sources. Similarly, for TDOA systems, the changes in speed of sound across the distorted paths cause the wave front to arrive earlier or later than that of an assumed perfect mach cone. Both TOA and DOA systems will suffer from the effects of atmospheric refraction, which are difficult to predict. It can only be said, generally, that as the temperature and/or wind conditions become increasingly severe the location errors will continue to grow. Preliminary analysis has been conducted using simple models of acoustic refraction due to sharp gradients in wind or temperature. The associated refraction angles and changes in arrival times can easily exceed the instrumentation-limited errors described above. Further discussion of errors induced by atmospheric gradient refraction is beyond the scope of this paper.

Discussion and Conclusions

In summary, we have developed a generalized mathematical approach to locate a segment of a projectile trajectory state vector from taking measurements of its bow wave properties, particularly either time of arrival or direction of arrival. The method can be employed to model and design a measurement space in which the sensors can theoretically be placed at arbitrary positions. The projectile can enter the space at an arbitrary location and direction. Illustrations, using simpler special case configurations, have been described. Of concern is the attainable accuracy of projectile location. The accuracy will of course depend on the scale of the measurement space and the design of the sensor network to meet the requirements of the application. In the examples shown, we estimated the degree of precision that would be required by either TOA measurements or DOA measurements to attain location accuracies on the order of tens of millimeters to hundreds of millimeters over measurement spaces on the order of 5 by 5 meters. To attain these levels of accuracy, the instrumentation must provide TDOA precision on the order of a few microseconds or better. Angular measurements must be precise to less than 1 degree. We see that the accuracy will depend on the geometry of locating the sensors relative to the projectile. Both TOA and DOA systems have peculiar geometries, which increase or decrease positional sensitivity to measurement precision. The instrumentation precisions mentioned here, on the order of 1 microsecond and/or 1 degree, are easily attainable with well-designed equipment using microphonic transducers of adequate bandwidth when exposed to the high signal to noise ratio and uniquely sharp wavefronts of the leading edge of the bow wave. The methods described here should be applicable to both small and large caliber incoming rounds.

For these systems, instrumentation error is not the limiting factor. The major eventual limitation in real world, open air, outdoor weather conditions will be due to refraction effects induced by atmospheric gradients in temperature and/or strong turbulent wind gradients. Unfortunately, these gradients tend to most severe in very close (less than 1meter) to the ground. The gradients are not only a function of weather and solar loading but are influenced by the shape of the terrain and man made structures (walls, troughs, vehicles, etc) surrounding the immediate environment of the measurement apparatus.